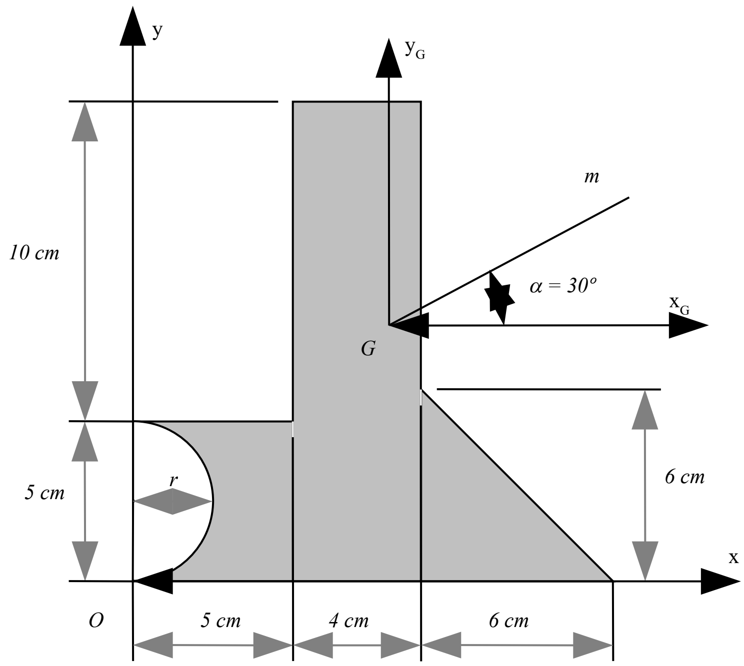
Ejercicio Nº3 - Enunciado

Dada la figura indicada,



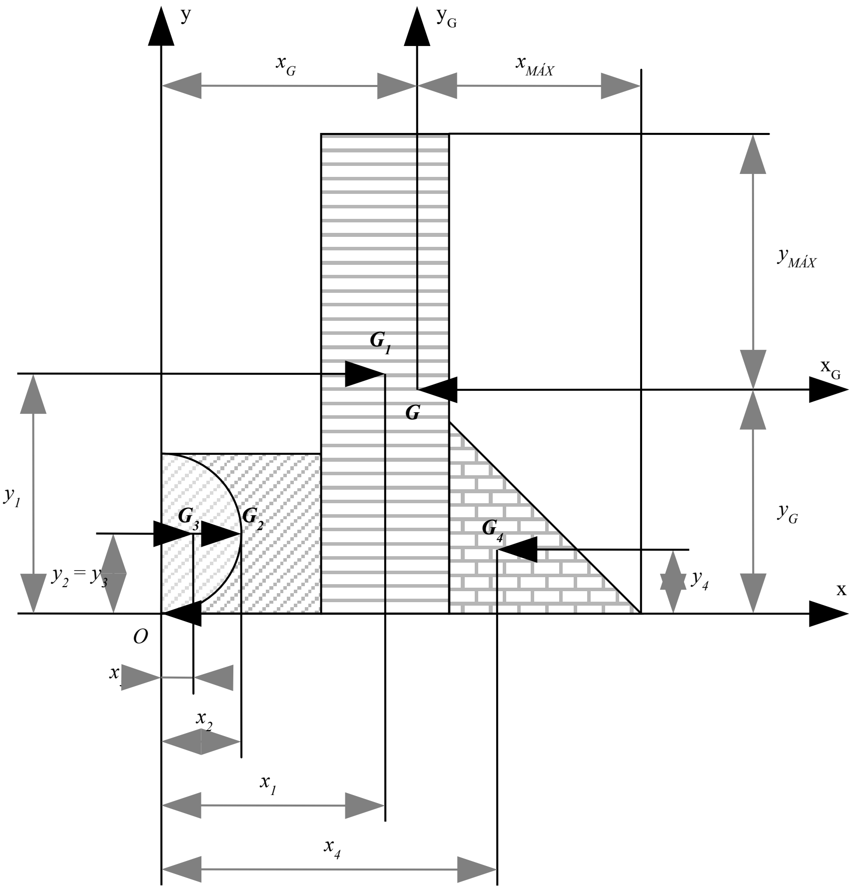
Se solicita determinar:

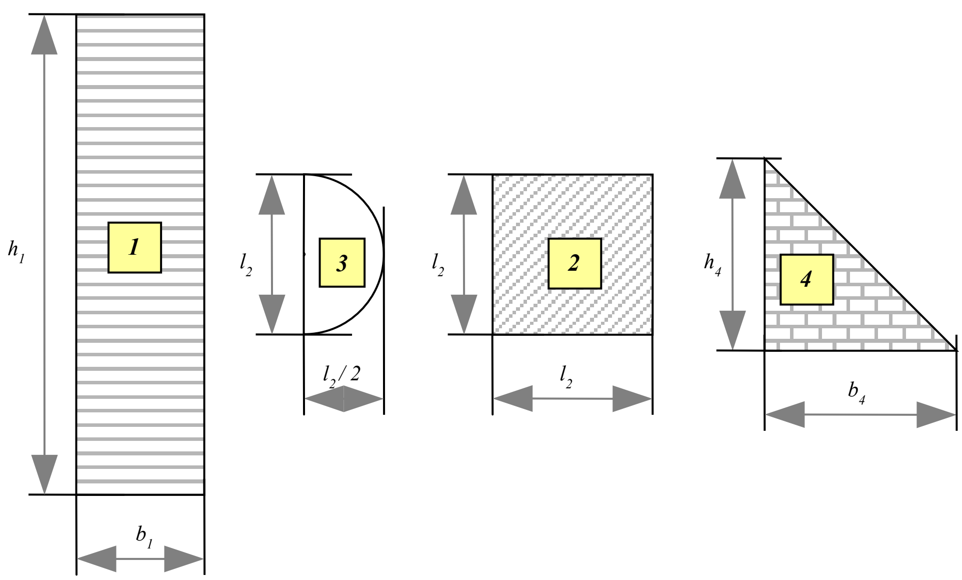
1. El baricentro de la misma
2. Los momentos de segundo orden, respecto de los ejes baricéntricos (J_{xG}, J_{yG}, J_{xGyG})
3. Los módulos resistentes, respecto de los ejes baricéntricos (W_{xG}, W_{yG})
4. Por el método analítico, la posición de los ejes principales de inercia (α_O) y sus respectivos valores J_I y J_{II}

Ejercicio Nº 3 – Resolución

1. Cálculo del baricentro

Se divide a la mismo en cuatro figuras, cuyos baricentos son conocidos





Áreas de cada una de las figuras formadas:

$$F_1 = b_1 \cdot h_1 = 15 \cdot 4 = 60 \cdot cm^2$$
$$F_2 = l_2{}^2 = 5^2 = 25 \cdot cm^2$$
$$F_3 = \frac{1}{2} \cdot \frac{\pi \cdot l_2{}^2}{4} = \frac{\pi \cdot 5^2}{8} = 9,818 \cdot cm^2$$
$$F_4 = \frac{b_4 \cdot h_4}{2} = \frac{6 \cdot 6}{2} = 18 \cdot cm^2$$

Distancias de los centros de gravedad de cada figura a los ejes coordenados *x* e *y*:

$$\left.\begin{aligned} x_1 &= \frac{b_1}{2} + l_2 = \frac{4}{2} + 5 = 7 \cdot cm \\ y_1 &= \frac{h_1}{2} = \frac{15}{2} = 7,5 \cdot cm \end{aligned}\right\} G_1\left(7 \quad 7,5\right)$$

$$\left.\begin{aligned} x_2 &= \frac{l_2}{2} = \frac{5}{2} = 2,5 \cdot cm \\ y_2 &= \frac{l_2}{2} = \frac{5}{2} = 2,5 \cdot cm \end{aligned}\right\} G_2\left(2,5 \quad 2,5\right)$$

$$\left.\begin{aligned} x_3 &= \frac{4 \cdot l_2}{6 \cdot \pi} = \frac{4 \cdot 5}{6 \cdot \pi} = 1,061 \cdot cm \\ y_3 &= \frac{l_2}{2} = \frac{5}{2} = 2,5 \cdot cm \end{aligned}\right\} G_3\left(1,061 \quad 2,5\right)$$

$$\left.\begin{aligned} x_4 &= \frac{b_4}{3} + l_2 + b_1 = \frac{6}{3} + 5 + 4 = 11 \cdot cm \\ y_4 &= \frac{h_4}{3} = \frac{6}{3} = 2 \cdot cm \end{aligned}\right\} G_4\left(11 \quad 2\right)$$

Área total de la figura *F*:

$$F = F_1 + F_2 - F_3 + F_4 = 60 + 25 - 9.818 + 18$$
$$\boldsymbol{F = 93,182 \cdot cm^2}$$

Posición del centro de gravedad *G*:

$$x_G = \frac{\sum_{i=1}^{4} F_i \cdot x_i}{\sum_{i=1}^{4} F_i} = \frac{F_1 \cdot x_1 + F_2 \cdot x_2 - F_3 \cdot x_3 + F_4 \cdot x_4}{F}$$
$$x_G = \frac{60 \cdot 7 + 25 \cdot 2,5 - 9,818 \cdot 1,061 + 18 \cdot 11}{93,182}$$
$$\boldsymbol{x_G = 7,191 \cdot cm}$$

$$y_G = \frac{\sum_{i=1}^{4} F_i \cdot y_i}{\sum_{i=1}^{4} F_i} = \frac{F_1 \cdot y_1 + F_2 \cdot y_2 - F_3 \cdot y_3 + F_4 \cdot y_4}{F}$$
$$y_G = \frac{60 \cdot 7,5 + 25 \cdot 2,5 - 9,818 \cdot 2,5 + 18 \cdot 2}{93,182}$$
$$\boldsymbol{y_G = 5,623 \cdot cm}$$

2. Determinación de los momentos de segundo orden, respecto de los ejes baricéntricos (J_{xG}, J_{yG}, J_{xGyG})

Distancias de los centros de gravedad de cada figura G_i al centro de gravedad de la figura total G:

$$x_{1G} = x_1 - x_G = 7 - 7,191 = -0,191 \cdot cm$$
$$x_{2G} = x_2 - x_G = 2,5 - 7,191 = -4,691 \cdot cm$$
$$x_{3G} = x_3 - x_G = 1,061 - 7,191 = -6,130 \cdot cm$$
$$x_{4G} = x_4 - x_G = 11 - 7,191 = 3,809 \cdot cm$$

$$y_{1G} = y_1 - y_G = 7,5 - 5,623 = 1,877 \cdot cm$$
$$y_{2G} = y_2 - y_G = 2,5 - 5,623 = -3,123 \cdot cm$$
$$y_{3G} = y_3 - y_G = 2,5 - 5,623 = -3,123 \cdot cm$$
$$y_{4G} = y_4 - y_G = 2 - 5,623 = -3,623 \cdot cm$$

Momentos de inercia baricéntricos de cada figura

$$J_{xG_1} = \frac{b_1 \cdot h_1^3}{12} = \frac{4 \cdot 15^3}{12} = 1125,000 \cdot cm^4$$
$$J_{xG_2} = \frac{l_2^4}{12} = \frac{5^4}{12} = 52,083 \cdot cm^4$$
$$J_{xG_3} = \frac{\frac{\pi \cdot l_2^4}{64}}{2} = \frac{\frac{\pi \cdot 5^4}{64}}{2} = 15,340 \cdot cm^4$$
$$J_{xG_4} = \frac{b_4 \cdot h_4^3}{36} = \frac{6 \cdot 6^3}{36} = 36,000 \cdot cm^4$$

$$J_{yG_1} = \frac{h_1 \cdot b_1^3}{12} = \frac{15 \cdot 4^3}{12} = 80,000 \cdot cm^4$$
$$J_{yG_2} = \frac{l_2^4}{12} = \frac{5^4}{12} = 52,083 \cdot cm^4$$
$$J_{yG_3} = \frac{\pi \cdot \left(\frac{l_2}{2}\right)^4}{8} - \frac{8 \cdot \left(\frac{l_2}{2}\right)^4}{9 \cdot \pi} = \frac{\pi \cdot \left(\frac{5}{2}\right)^4}{8} - \frac{8 \cdot \left(\frac{5}{2}\right)^4}{9 \cdot \pi} = 15,340 - 11,052 = 4,287 \cdot cm^4$$
$$J_{yG_4} = \frac{h_4 \cdot b_4^3}{36} = \frac{6 \cdot 6^3}{36} = 36,000 \cdot cm^4$$

$$J_{xGyG_1} = J_{xGyG_2} = J_{xGyG_3} = 0 \cdot cm^4$$
$$J_{xGyG_4} = -\frac{b_4^2 \cdot h_4^2}{72} = -\frac{6^2 \cdot 6^2}{72} = -18,000 \cdot cm^4$$

Momentos de inercia baricéntricos de la figura total

$$J_{xG} = \sum_{i=1}^{4} \left(J_{xG_i} + F_i \cdot y_{iG}^2 \right)$$
$$J_{xG} = J_{xG_1} + F_1 \cdot y_{1G}^2 + J_{xG_2} + F_2 \cdot y_{2G}^2 - J_{xG_3} - F_3 \cdot y_{3G}^2 + J_{xG_4} + F_4 \cdot y_{4G}^2$$
$$J_{xG} = 1125 + 60 \cdot 1,877^2 + 52,083 + 25 \cdot (-3,123)^2 - 15,340 - 9,818 \cdot (-3,123)^2 + 36 + 18 \cdot (-3,623)^2$$

$$\boldsymbol{J_{xG} = 1793,476 \cdot cm^4}$$

$$J_{yG}=\sum_{i=1}^{4}\left(J_{yG_i}+F_i\cdot x_{iG}{}^2\right)$$
$$J_{yG}=J_{yG_1}+F_1\cdot x_{1G}{}^2+J_{yG_2}+F_2\cdot x_{2G}{}^2-J_{yG_3}-F_3\cdot x_{3G}{}^2+J_{yG_4}+F_4\cdot x_{4G}{}^2$$
$$J_{xG}=80+60\cdot(-0{,}191)^2+52{,}083+25\cdot(-4{,}691)^2-4{,}287-9{,}818\cdot(-6{,}130)^2+36+18\cdot 3{,}809^2$$

$$\boldsymbol{J_{yG}=608{,}356\cdot cm^4}$$

$$J_{xGyG}=\sum_{i=1}^{4}\left(J_{xGyG_i}+F_i\cdot x_{iG}\cdot y_{iG}\right)$$
$$J_{xGyG}=F_1\cdot x_{1G}\cdot y_{1G}+F_2\cdot x_{2G}\cdot y_{2G}-F_3\cdot x_{3G}\cdot y_{3G}+J_{xGyG_4}+F_4\cdot x_{4G}\cdot y_{4G}$$
$$J_{xGyG}=60\cdot(-0{,}191)\cdot 1{,}877+25\cdot(-4{,}691)\cdot(-3{,}123)-9{,}818\cdot(-6{,}130)\cdot(-3{,}123)+(-18)+18\cdot 3{,}809\cdot(-3{,}623)4$$

$$\boldsymbol{J_{xGyG}=-109{,}616\cdot cm^4}$$

3. Determinación de los módulos resistentes, respecto de los ejes baricéntricos (W_{xG}, W_{yG})

$$W_{xG}=\frac{J_{xG}}{y_{máx}}=\frac{J_{xG}}{h_1-y_G}=\frac{1793{,}476}{15-5{,}623}\qquad \boldsymbol{W_{xG}=191{,}3\cdot cm^3}$$

$$W_{yG}=\frac{J_{yG}}{x_{máx}}=\frac{J_{yG}}{b_1+l_2+b_4-x_G}=\frac{608{,}356}{4+5+6-7{,}191}\qquad \boldsymbol{W_{yG}=77{,}9\cdot cm^3}$$

4. Determinación analítica de la posición de los ejes principales de inercia (α_O) y sus respectivos valores J_I y J_{II}

Cálculo de α_O

$$\tan(2\cdot\alpha_O)=\frac{2\cdot J_{xGyG}}{J_{yG}-J_{xG}}$$
$$\alpha_O=\frac{1}{2}\cdot\tan^{-1}\left(\frac{2\cdot J_{xGyG}}{J_{yG}-J_{xG}}\right)=\frac{1}{2}\cdot\tan^{-1}\left(\frac{2\cdot(-109{,}616)}{608{,}356-1793{,}476}\right)=\frac{1}{2}\cdot\tan^{-1}(0{,}185)$$

$$\boldsymbol{\alpha_O=5^{\circ}\quad 14'}$$

Cálculo de J_I y J_{II}

$$J_{I,II}=\frac{1}{2}\cdot\left(J_{xG}+J_{yG}\right)\pm\frac{1}{2}\cdot\sqrt{\left(J_{xG}-J_{yG}\right)^2+4\cdot J_{xGyG}{}^2}$$
$$J_{I,II}=\frac{1}{2}\cdot(1793{,}48+608{,}36)\pm\frac{1}{2}\cdot\sqrt{(1793{,}48-608{,}36)^2+4\cdot(-109{,}616)^2}$$
$$J_{I,II}=1200{,}92\pm 602{,}61$$

$$\boldsymbol{J_I=1803{,}53\cdot cm^4}$$
$$\boldsymbol{J_{II}=598{,}31\cdot cm^4}$$

Observar que se verifica que:

$$J_I+J_{II}=J_{xG}+J_{yG}$$
$$J_I+J_{II}=1803{,}53+598.31=2401{,}84\cdot cm^4$$
$$J_{xG}+J_{yG}=1793{,}476+608{,}356=2401{,}84\cdot cm^4$$